4.3 Arithmetic [0NN]

We will define the addition operation between natural numbers, formally

$$\cdot + \cdot : \mathbb{N}\times\mathbb{N}\to\mathbb{N} \quad , \quad (h,k)\mapsto h+k \quad .$$

Definition 4.11. *[292]*

This operation is commutative and associative, as shown below.
Note that $h+0=f_h(0)=h$ (basis of recursion); also $0+n=f_0(n)=n$ (shows easily by induction).
To prove that it is commutative, we first show that

Lemma 4.12. *[27N]*

Proposition 4.13. *[27P]*

At this point we can give a name to $1=S(0)$ and notice that $S(n)=n+1.$ So from now on we could do without the symbol S.
With similar procedures we show that addition is associative.

Proposition 4.14. *[27Q]*

Multiplication is similarly defined.

Definition 4.15. *[28V]*

then we can prove the known properties (commutativity, associativity, distributivity).

Exercises

E4.16 [27R]

E4.17 [27S]

E4.18 [27V]

E4.19 [27W]

E4.20 [27X]

E4.21 [28T]

E4.22 [281]

E4.23 [27Z]

E4.24 [280]

In the following we will simply write nm instead of $n\times m$.