Definition 10.b.3. [ONX] *For the following exercises we define that*

1. *a set E is **open** if*

$$\forall x_0 \in E, \exists r > 0 : B(x_0, r) \subseteq E \quad . \tag{10.b.4}$$

It is easily seen that $\emptyset, X$ are open sets; that the intersection of a finite number of open sets is an open set; that the union of an arbitrary number of open set is an open set. So these open sets form a topology.

2. *The **interior** E° of a set E is*

$$E^\circ = \{x \in E : \exists r > 0, B_r(x) \subseteq E\} ; \tag{10.b.5}$$

It is easy to verify that $E^\circ \subseteq E$, and that E is open if and only if $E^\circ = E$ (exercise [OPB]*).*

3. *A set is **closed** if the complement is open.*

4. *A point $x_0 \in X$ is **adherent** to E if*

$$\forall r > 0 , \quad E \cap B_r(x_0) \neq \emptyset \quad .$$

5. *The **closure** $\overline{E}$ of E is the set of adherent points; it is easy to verify that $E \subseteq \overline{E}$; It is shown that $\overline{E} = E$ if and only if E is closed (exercise* [OPM]*).*

6. *A is **dense in** B if $\overline{A} \supseteq B$, that is, if for every $x \in B$ and for every $r > 0$ the intersection $B_r(x) \cap A$ is not empty.*