

## Exercises

E10.41 [OPS] Prerequisites: [OM7]. Let  $X$  be a set with two distances  $d_1, d_2$ ; let's call  $\tau_1, \tau_2$  respectively the induced topologies. We have that  $\tau_1 \subseteq \tau_2$  if and only if

$$\forall x \in X \forall r_1 > 0 \exists r_2 > 0 : B^2(x, r_2) \subseteq B^1(x, r_1)$$

where

$$B^2(x, r_2) = \{y \in X : d^2(x, y) < r_2\} \quad , \quad B^1(x, r_1) = \{y \in X : d^1(x, y) < r_1\}$$

*Note that this exercise is the analogue in metric spaces of the principle [OM7] for the bases of topologies.*