## Exercises

E9.41 [0PT] Prerequisites: [0M3], [0PS], [107], [10F], [10J].

Having fixed $(X_1, d_1), \ldots, (X_n, d_n)$ metric spaces, let $X = X_1 \times \cdots \times X_n$. Let $\varphi$ be one of the norms defined in eqn. [(11.14)] in Sec. [2CK]. Two possible examples are $\varphi(x) = |x_1| + \cdots + |x_n|$ or $\varphi(x) = \max_{i=1\ldots n} |x_i|$. Finally, let's define for $x, y \in X$

$$d(x, y) = \varphi\big(d_1(x_1, y_1), \ldots, d_n(x_n, y_n)\big) \quad . \tag{9.42}$$

Show that $d$ is a distance; show that the topology in $(X, d)$ coincides with the product topology (see [0M3]).

*Note that this approach generalizes the definition of the Euclidean distance between points in $\mathbb{R}^n$ (taking $X_i = \mathbb{R}$ and $\varphi(z) = \sqrt{\sum_i |z_i|^2}$). We deduce that the topology of $\mathbb{R}^n$ is the product of the topologies of $\mathbb{R}$.*

**Solution 1.** *[0PX]*

See also the exercise [0QM], which reformulates the above using the concept of *bases of topologies*.

[ [0PV] ] [ [0PW] ]