Exercises

E10.b.24 [0PT] Prerequisites: [0M3], [0PS], [107], [10F], [10J].

Having fixed $(X_1, d_1), \ldots, (X_n, d_n)$ metric spaces, let $X = X_1 \times \cdots \times X_n$.

Let φ be one of the norms defined in eqn. [2CK] in Sec. [2CK]. Two possible examples are $\varphi(x) = |x_1| + \cdots + |x_n|$ or $\varphi(x) = \max_{i=1\ldots n} |x_i|$.

Finally, let's define for $x, y \in X$

$$d(x,y) = \varphi\big(d_1(x_1,y_1), \ldots, d_n(x_n,y_n)\big) \quad . \tag{10.b.25}$$

Show that d is a distance; show that the topology in (X, d) coincides with the product topology (see [0M3]).

Note that this approach generalizes the definition of the Euclidean distance between points in $\mathbb{R}^n$ (taking $X_i = \mathbb{R}$ and $\varphi(z) = \sqrt{\sum_i |z_i|^2}$). We deduce that the topology of $\mathbb{R}^n$ is the product of the topologies of $\mathbb{R}$.

Solution 1. *[0PX]*

See also the exercise [0QM], which reformulates the above using the concept of *bases of topologies*.

[[0PV]] [[0PW]]