## Exercises

E9.27 [OQO] Prerequisites: [OQ3], [OP3], [OPY], [OPN]. Let $r > 0$.

Let $D(x,r) \stackrel{\text{def}}{=} \{y \in X : d(x,y) \leq r\}$ be the disk; show that $\overline{B(x,r)} \subseteq D(x,r)$ and that $B(x,r) \subseteq D(x,r)^\circ$.

Let $S(x,r) \stackrel{\text{def}}{=} \{y \in X : d(x,y) = r\}$ be the sphere; show that $\partial B(x,r) \subseteq S(x,r)$.

Find examples of metric spaces in which the above equalities (one, or both) do not hold.

Find an example of a metric space where there is a disk that is open[a].

(See also [OSM] for the case of space $\mathbb{R}^n$).

**Solution 1.** *[OQ1]*

[a] There are also spaces where every ball is closed, see [OQF].