Exercises

E9.27 [OQC] Prerequisites:[OR9]. Show that, for every closed set $C \subseteq X$ there exist countably many open sets A_n such that $\bigcap_n A_n = C$.

Solution 1. *[OQD]*

A set obtained as an intersection of countably many open sets is known as "a G_δ set". The previous exercise shows that in a metric space every closed is a G_δ.

Passing to the complement set, one obtains this statement. A set that is union of countably many closed sets is known as "an F_σ set". The previous exercise shows that in a metric space every open set is an F_σ set.

See also the section [14J].