Exercises

E9.57 [OR2] Suppose that d satisfies all distance requirements except *"separation property"*. Consider the relation $\sim$ on X defined as $x \sim y \iff d(x,y) = 0$; show that is an equivalence relation. Let's define $Y = X/\sim$; show that the function d "passes to the quotient", that is, there exists $\tilde{d} : Y \times Y \to [0,\infty)$ such that, for every choice of classes $s,t \in Y$ and every choice of $x \in s, y \in t$ you have $\tilde{d}(s,t) = d(x,y)$. Finally, show that $\tilde{d}$ is a distance on Y.

This procedure is the metric space equivalent of *Kolmogoroff quotient*.