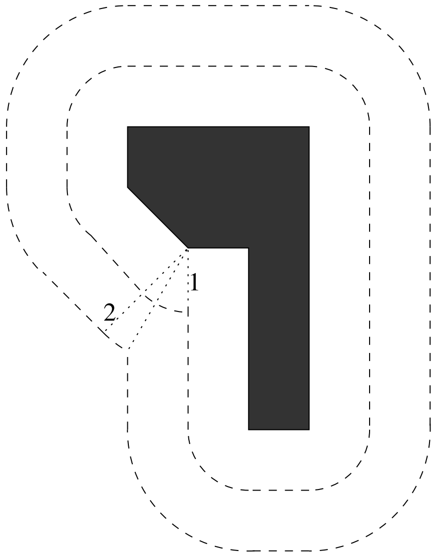

Figure 2: Fattening of a set; exercise E9.62

Exercises

E9.62 `Topics:fattened set.Prerequisites:[0R9].` [0RC]

Consider a metric space (M, d). Let $A \subseteq M$ be closed and non-empty, let $r > 0$ be fixed, and let d_A be the distance function defined as in eqn. [(9.62)]. Let then $E = \{x, d_A(x) \le r\}$, notice that it is closed.

(a) Show that
$$d_E(x) \ge \max\{0, (d_A(x) - r)\} \ . \qquad (9.62)$$

(b) Show that in (9.62) you have equality if $M = \mathbb{R}^N$.

(c) Give a simple example of a metric space where equality in (9.62) does not hold.

(d) If $M = \mathbb{R}^n$, given $A \subset \mathbb{R}^n$ closed non-empty, show that $E = A \oplus D_r$ where $D_r \stackrel{\text{def}}{=} \{x, |x| \le r\}$ and
$$A \oplus B \stackrel{\text{def}}{=} \{x + y, x \in A, y \in B\}$$
is the *Minkowski sum* of the two sets (see also Section [2CP]).

Solution 1. *[0RD]*

The set $\{x, d_A(x) \le r\} = A \oplus D_r$ is sometimes called the *"fattening"* of A. In figure 2 we see an example of a set A fattened to $r = 1, 2$; the set A is the black polygon (and is filled in), whereas the dashed lines in the drawing are the contours of the fattened sets. [75] See also the properties in sections [2CP] and [2CQ].

[75] The fattened sets are not drawn filled — otherwise they would cover A.

DRAFT