Exercises

E10.f.9 [OSG]Prerequisites:[OPJ], [OPQ], [OGJ], [OQ8].Difficulty:**.

Find a subset A of $\mathbb{R}$ such that the following 7 subsets of $\mathbb{R}$ are all distinct:

$$A,\ \overline{A},\ A^{\circ},\ \left(\overline{A}\right)^{\circ},\ \overline{(A^{\circ})},\ \overline{\left(\left(\overline{A}\right)^{\circ}\right)},\ \left(\overline{(A^{\circ})}\right)^{\circ}\ .$$

Also prove that no other different sets can be created by continuing in the same way (also replacing $\mathbb{R}$ with a generic metric space).

Solution 1. *[OSH]*