Exercises

E10.g.2 [OSM] Prerequisites: [OQO]. Let $B(x,r) \stackrel{\text{def}}{=} \{y \in \mathbb{R}^n : |x-y| < r\}$ be the ball; let $D(x,r) \stackrel{\text{def}}{=} \{y \in \mathbb{R}^n : |x-y| \leq r\}$ the disc; let $S(x,r) \stackrel{\text{def}}{=} \{y \in \mathbb{R}^n : |x-y| = r\}$ be the sphere. Show that $\overline{B(x,r)} = D(x,r)$, that $B(x,r) = D(x,r)^\circ$, and that $\partial B(x,r) = S(x,r)$. Also show that $B(x,r)$ is not closed and $D(x,r)$ is not open.

(This result holds more generally in any normed space: see [106]).