

## Exercises

E9.102 [OTH] Prerequisites: [OVP]. Let  $(X, d)$  be a compact metric space, and let  $f : X \rightarrow X$  be such that

$$\forall x, y \in X, x \neq y \Rightarrow d(f(x), f(y)) < d(x, y) \quad .$$

Show that  $f$  has a single fixed point  $\bar{x}$ .

Let  $x_0 \in X$  and define  $x_{n+1} = f(x_n)$  by recurrence: show that  $\lim_n x_n = \bar{x}$ .

This result is sometimes called *Edelstein's Theorem*.

**Solution 1.** [27C]