Theorem 9.101. [0V3] *Given a metric space (X, d) and its subset $C \subseteq X$, The following three conditions are equivalent.*

- *C is* sequentially compact*: every sequence $(x_n) \subset C$ has a subsequence converging to an element of C.*
- *C is* compact*: from each family of open sets whose union covers C, we can choose a finite subfamily whose union covers C.*
- *C is complete, and is* totally bounded*: for every $\varepsilon > 0$ there are finite points $x_1 ... x_n \in C$ such that $C \subseteq \bigcup_{i=1}^n B(x_i, \varepsilon)$.*