Exercises

9.101 [OVD]Let be given a metric space (X,d). As in [ONW] we define the *disk* $D(x,\varepsilon) \stackrel{\text{def}}{=} \{y \in X, d(x,y) \le \varepsilon\}$ (which is closed). (X,d) is *locally compact* if for every $x \in X$ there exists $\varepsilon > 0$ such that $D(x,\varepsilon)$ is compact. Consider this proposition.

«***Proposition*** *A locally compact metric space is complete.* ***Proof*** *Let $(x_n)_n \subset X$ be a Cauchy sequence, then eventually its terms are distant at most ε, so they are contained in a small compact disk, so there is a subsequence that converges, and then, by the result [ON8], the whole sequence converges. q.e.d.* »

If you think the proposition is true, rewrite the proof rigorously. If you think it's false, find a counterexample.

Solution 1. *[OVF]*