

Exercises

9.101 [OVT] Prerequisites: [OVS]. Let $X = C^0([0, 1])$ be the space of continuous and bounded functions $f : [0, 1] \rightarrow \mathbb{R}$, endowed with the usual distance

$$d_\infty(f, g) = \|f - g\|_\infty = \sup_{x \in [0, 1]} |f(x) - g(x)| \quad .$$

We know that (X, d_∞) is a complete metric space. Let

$$D(0, 1) = \{f \in X : d_\infty(0, f) \leq 1\} = \{f \in X : \forall x \in [0, 1], \quad |f(x)| \leq 1\}$$

the disk of center 0 (the function identically zero) and radius 1. We know from [OPY] that it is closed, and therefore it is complete. Show that D is not totally bounded by finding a sequence $(f_n) \subseteq D$ as explained in [OVS].