Exercises

9.121 [OWG]Prerequisites:[OWD]. We want to define a distance for the space of sequences. We proceed as in [OW9]. We choose $X_i = \mathbb{R}$ for each i and set that d_i is the Euclidean distance; then for $f, g : \mathbb{N} \to \mathbb{R}$ we define

$$d(f,g) = \sum_k 2^{-k} \varphi(|f(k) - g(k)|) \, .$$

We have constructed a metric space of sequences $(\mathbb{R}^{\mathbb{N}}, d)$.

In the space of sequences $(\mathbb{R}^{\mathbb{N}}, d)$ we define

$$K = \{f \in \mathbb{R}^{\mathbb{N}}, \forall k, |f(k)| \leq 1\} \quad .$$

Show that K is compact.

Solution 1. *[OWH]*