Definition 9.153. [0XF] *Each rational number $x \neq 0$ breaks down uniquely as a product*

$$x = \pm p_1^{m_1} p_2^{m_2} \cdots p_k^{m_k} \,, \tag{9.154}$$

*where $p_1 < p_2 < \cdots < p_k$ are prime numbers and the m_j non-null integers. Fixed as above a prime number p, we define the p–**adic absolute value** of $x \in \mathbb{Q}$ as*

$$|x|_p = \begin{cases} 0 & \text{if} \quad x = 0 \\ p^{-m} & \text{if } p^m \text{ is the factor with base } p \text{ in the decomposition (9.1} \end{cases}$$

*Finally, we define $d(x, y) = |x - y|_p$, which will turn out to be a distance on $\mathbb{Q}$, called p–**adic distance**.*