Definition 9.144. [0XF] *Each rational number* $x \neq 0$ *breaks down uniquely as a product*

$$x = \pm p_1^{m_1} p_2^{m_2} \cdots p_k^{m_k}\ , \tag{9.145}$$

where $p_1 < p_2 < \cdots < p_k$ *are prime numbers and the* m_j *integers. Fixed as above a prime number* p*, we define the* p***–adic absolute value*** *of* $x \in \mathbb{Q}$ *as*

$$|x|_p = \begin{cases} 0 & \text{if} \quad x = 0 \\ p^{-m} & \text{if } p^m \text{ is the factor with base } p \text{ in the decomposition} \end{cases} \quad (9.1$$

Finally, we define $d(x, y) = |x - y|_p$*, which will turn out to be a distance on* $\mathbb{Q}$*, called* p***–adic distance****.*