Exercises

9.146 [0XH] Prove these fundamental relation.

(a) $|1|_p = 1$ and more generally $|n|_p \leq 1$ for every nonnull integer n, with equality if n is not divisible by p.

(b) Given n nonnull integer, we have that $|n|_p = p^{-\varphi_p(n)}$.

(c) Given n, m integers, we have that $\varphi_p(n+m) \geq \min\{\varphi_p(n), \varphi_p(m)\}$ with equality if $\varphi_p(n) \neq \varphi_p(m)$.

(d) Given n, m nonzero integers, we have that $\varphi_p(nm) = \varphi_p(n) + \varphi_p(m)$ and therefore $|nm|_p = |n|_p |m|_p$.

(e) Given $x = a/b$ with a, b nonnull integers we have that $|x|_p = p^{-\varphi_p(a)+\varphi_p(b)}$. Note that if a, b are coprime, then one of the two terms $\varphi_p(a), \varphi_p(b)$ is zero.

(f) Prove that $|xy|_p = |x|_p |y|_p$ for $x, y \in \mathbb{Q}$.

(g) Prove that $|x/y|_p = |x|_p / |y|_p$ for $x, y \in \mathbb{Q}$ nonzero.

[[0XJ]] [[0XK]]