Exercises

9.146 [OXM] Check that

$$|x+y|_p \le \max\left\{|x|_p, |y|_p\right\} \qquad (9.146)$$

for each $x,\ y \in \mathbb{Q}$. and therefore

$$d_p(x,z) \le \max\left\{d_p(x,y), d_p(y,z)\right\}, \qquad \forall\, x,\ y,\ z \in \mathbb{Q}\ .$$

that is, this is an ultrametric (and therefore a distance).

Solution 1. *[OXN]*

The properties [E9.146f] and (9.146) say that the *p-adic valuation* is an absolute value, and indeed it is a *Krull valuation*.

[[OXP]]