**Exercises**

9.155 [0Y7] Through this bijection we transport the Euclidean distance from $S^1$ to $\mathbb{R}/2\pi$ defining

$$d_e([s],[t]) = |\Phi([s]) - \Phi([t])|_{\mathbb{R}^2} \ .$$

With this choice the map $\Phi$ turns out to be an isometry between $(S^1, d)$ and $(\mathbb{R}/2\pi, d_e)$ (see the Definition [0TK]). So the latter is a complete metric space.

With some simple calculations it can be deduced that

$$d_e([s],[t]) = \sqrt{|\cos(t) - \cos(s)|^2 + |\sin(t) - \sin(s)|^2} = \sqrt{2 - 2\,c$$

Then we define the function

$$d_a([s],[t]) = \inf\{|s - t - 2\pi k| \,:\, k \in \mathbb{Z}\} \ ,$$

show that it is a distance on $\mathbb{R}/2\pi$.

**Solution 1.** *[0Y8]*