

## 10 Dimension

[OYH]

Let  $(X, d)$  be a metric space. Let in the following  $K$  a compact non-empty subset of  $X$ , and  $N(\rho)$  the minimum number of balls of radius  $\rho$  that are needed to cover  $K$ .<sup>79</sup>

**Definition 10.1.** [OYJ]

If the limit does not exist, we can still use the limsup and the liminf to define the *upper and lower dimension*.

Note that this definition depends *a priori* on the choice of the distance, i.e.  $N = N(\rho, K, d)$  and  $\dim = \dim(K, d)$ . See in particular [OYZ].

### Exercises

E10.2 [OYK]

E10.3 [OYN]

E10.4 [OYQ]

E10.5 [OYS]

E10.6 [OYV]

E10.7 [OYX]

E10.8 [OYZ]

E10.9 [OZ1]

E10.10 [OZ3]

E10.11 [OZ7]

E10.12 [OZB]

E10.13 [OZD]

E10.14 [OZG]

E10.15 [OZJ]

E10.16 [OZM]

E10.17 [OZF]

E10.18 [OZR]

**QuasiEsercizio 41.** [OZS]

<sup>79</sup>By the Heine–Borel theorem [OV3] we know that  $N(\rho) < \infty$