Exercises

E10.4 [0Z3] We indicate an operating policy that can be used in the following exercises.

- If there is a descending sequence $\rho_j \to 0$ and h_j positive such that h_j balls of radious ρ_j are enough to cover K, then

$$\limsup_{\rho\to 0+} \frac{\log N(\rho)}{\log(1/\rho)} \le \limsup_{j\to\infty} \frac{\log h_{j+1}}{\log(1/\rho_j)} \ . \tag{10.4}$$

- If, on the other hand, there is a descending sequence $r_j \to 0$, and $C_n \subseteq K$ finite families of points that are at least distant r_j from each other, i.e. for which

$$\forall x, y \in C_n, x \neq y \Rightarrow d(x,y) \ge r_j \ , \tag{10.5}$$

then

$$\liminf_{\rho\to 0+} \frac{\log N(\rho)}{\log(1/\rho)} \ge \liminf_{j\to\infty} \frac{\log l_j}{\log(1/r_{j+1})} \ . \tag{10.6}$$

where $l_j = |C_j|$ is the cardinality of C_j. Note that the points of $x \in C_j$ are centers of disjoint balls $B(x, r_j/2)$, therefore $l_j \le P(r_j/2)$, as defined in [0YS].

In particular, if

$$\limsup_{j\to\infty} \frac{\log h_{j+1}}{\log(1/\rho_j)} = \liminf_{j\to\infty} \frac{\log l_j}{\log(1/r_{j+1})} = \beta \tag{10.7}$$

then the set K has dimension β.

[[0Z4]]

Solution 1. *[0Z5]*