Exercises

E10.18 [OZ7]Prerequisites: [10X] [OZ1] [OZ3].Difficulty:*.Let $K \subseteq \mathbb{R}^m$ compact. Consider the family of closed cubes with edge length 2^{-n} and centers at the grid points $2^{-n}\mathbb{Z}^m$. We call it *"n-tessellation"*. Let N_n be the number of cubes of the *n-tessellation* intersecting K. Show that N_n is weakly increasing. Show that the following limit exists

$$\lim_{n\to\infty} \frac{\log_2 N_n}{n} \tag{10.19}$$

if and only if the limit [(10.3)] (that defines the dimension) exists. Show that, when they both exist, they coincide. This approach to computing the dimension is called *Box Dimension*.

Solution 1. *[OZ8]*

These quantities have an interpretation in rate-distortion theory. "n" is the position of the last significant digit (in base 2) in determining the position of a point x. "$\log_2 N_n$*" is the number of "bits" needed to identify any* $x \in K$ *with such precision.*