Definition 12.0.2. [OZV] *A* norm *is an operation that maps a vector $v \in X$ in a real number $\|v\|$, which satisfies*

1. $\|v\| \geq 0$ *and* $\|v\| = 0$ *if and only if* $v = 0$*;*

2. *for every* $v \in X$ *and* $t \in \mathbb{R}$ *we have* $|t| \, \|v\| = \|tv\|$ *(we will say that the norm is* absolutely homogeneous*);*

3. (Triangle inequality) *for every* $v, w \in X$ *we have*

$$\|v + w\| \leq \|v\| + \|w\| \quad ;$$

this says that one side of a triangle is less than the sum of the other two.