Exercises

E11.4 [0ZX] Let X be a vector space and $f : V \to \mathbb{R}$ a function that is *positively homogeneous*, that is: for every $v \in X$ and $t \geq 0$ you have $tf(v) = f(tv)$.

Show that f is convex if and only if the *triangle inequality* holds: for every $v, w \in X$ you have

$$f(v + w) \leq f(v) + f(w) \quad .$$

In particular, a norm is always a convex function.