

Exercises

E11.3 [0ZZ] Prerequisites: [106], [19D], [0ZX]. Difficulty:*. We will say that the normed space $(X, \|\cdot\|)$ is *strictly convex*^a if the following equivalent properties apply.

- The disc $D = \{x \in X : \|x\| \leq 1\}$ is strictly convex.^b
- The sphere $\{x \in X, \|x\| = 1\}$ does not contain non-trivial segments (that is, segments of positive length).
- For $v, w \in D$ with $\|v\| = \|w\| = 1$ and $v \neq w$, for every t such that $0 < t < 1$, we have that $\|tv + (1 - t)w\| < 1$.
- For every $v, w \in X$ that are linearly independent we have $\|v + w\| < \|v\| + \|w\|$.

Show that the previous four clauses are equivalent.

[[101]]

Solution 1. [102]

^aSee [21] for more properties.

^bThe definition is in [19D].