Exercises

E11.3 [106] Prerequisites:[0Q0].

Let again X be a normed vector space with norm $\|\cdot\|$. Let $B(x,r) \stackrel{\text{def}}{=} \{y \in X : \|x-y\| < r\}$ be the ball. Let $D(x,r) \stackrel{\text{def}}{=} \{y \in X : \|x-y\| \le r\}$ be the disk. Let $S(x,r) \stackrel{\text{def}}{=} \{y \in X : \|x-y\| = r\}$ be the sphere. Show that $\overline{B(x,r)} = D(x,r)$, that $B(x,r) = D(x,r)^\circ$, and that $\partial B(x,r) = \partial D(x,r) = S(x,r)$. Also show that $B(x,r)$ is not closed and $D(x,r)$ is not open.