Exercises

E11.4 [109] *We want to show that "the norms in $\mathbb{R}^n$ are all equivalent."*

Let $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$ be the Euclidean norm. Let $\phi : \mathbb{R}^n \to [0, \infty)$ be a norm: it can be shown that ϕ is a convex function, see [0ZX]; and therefore ϕ is a continuous function, see [186]. Use this fact to prove that there exist $0 < a < b$ such that

$$\forall x, \quad a\|x\| \leq \phi(x) \leq b\|x\| \ . \tag{11.4}$$

Solution 1. *[10B]*