Exercises

E11.16 [10F] Prerequisites: [1H8]. Having fixed $t, s \in [1, \infty]$ with $s > t$ and $x \in \mathbb{R}^n$, show that $\|x\|_s \leq \|x\|_t$. Also show that $\|x\|_s < \|x\|_t$ if $n \geq 2$ and $x \neq 0$ and x is not a multiple of one of the vectors of the canonical basis $e_1, \ldots e_n$.

Hints:

1. *use that* $1 + t^p \leq (1 + t)^p$ *for* $p \geq 1$ *and* $t \geq 0$*; or*
2. *use Lagrange multipliers; or*
3. *remember that* $f(a + b) > f(a) + f(b)$ *when* $a \geq 0, b > 0$ $f(0) = 0$ *and* $f : [0, \infty) \to \mathbb{R}$ *is strictly convex and continuous in 0 (see exercise [192]), therefore derive* $\frac{d}{dt}(\log \|x\|_t)$ *and set* $f(z) = z \log(z)$*).*

Solution 1. [10G]

[[10H]]