Exercises

E11.20 [10M] Let be given $p, q \in [1, \infty]$ such that $1/p + 1/q = 1$ [a] and $x, y \in \mathbb{R}^n$; show the **Hölder inequality** in this form

$$\sum_{i=1}^{n} |x_i y_i| \leq \|x\|_p \|y\|_q \quad . \tag{11.21}$$

In what cases is there equality?

Tips: Fix $x_i, y_i \geq 0$. For the cases with $p, q < \infty$ you can:

- *use Young inequality ([194] or [1V7]);*
- *use Lagrange multipliers;*
- *start from the case $n = 2$ and set $x_2 = t x_1$ and $y_2 = a y_1$; then, for cases $n \geq 3$ use induction.*

Solution 1. [10N]

[a] This means that if $p = 1$ then $q = \infty$; and vice versa.