Exercises

11.20 [100] Prerequisites: [100]. Given $p \in [1, \infty]$ show the **Minkowski** inequality

$$\|x + y\|_{p} \le \|x\|_{p} + \|y\|_{p} \quad . \tag{11.20}$$

There follows that $||x||_p$ are norms.

For $p \in (1, \infty)$ find a simple condition (necessary and sufficient) that involves equality; compare it with [ozy]; deduce that \mathbb{R}^n , with the norm $\|\cdot\|_p$ for $p \in (1, \infty)$, is a *strictly convex normed space* (see [ozz]).

Solution 1. [10R]