Exercises

E11.42 [118] Topics:total convergence.Prerequisites:[ON8],[ONC],[ONF].

Let V be a vector space with a norm $\|x\|$; So V is also a metric space with the metric $d(x, y) = \|x - y\|$. Show that the following two clauses are equivalent.

- (V, d) is complete.
- For each sequence $(v_n)_n \subset V$ such that $\sum_n \|v_n\| < \infty$, the series $\sum_n v_n$ converges.

(The second is sometimes called the "total convergence criterion")

A normed vector space $(V, |\cdot\|)$ such that the associated metric space (V, d) is complete, is called a **Banach space**.

Solution 1. *[119]*