Exercises

E13.a.5 [13D] Let $f : X \to \mathbb{R}$; the following assertions are equivalent.

1. f is *lower semicontinuous*.
2. For every t, we have that the sublevel

$$S_t = \{x \in X, f(x) \leq t\}$$

   is closed.
3. The epigraph

$$E = \{(x, t) \in X \times \mathbb{R}, f(x) \leq t\}$$

   is closed in $X \times \mathbb{R}$.

Note that the second condition means that f is continuous from (X, τ) to $\mathbb{R}, \tau_+$ where $\tau_+ = \{(a, \infty) : a \in \mathbb{R}\} \cup \{\varnothing, \mathbb{R}\}$ is the set of half-lines, which is a topology (easy verification).

Then formulate the equivalent theorem for functions *upper semicontinuous*.

Solution 1. [13F]