

## Exercises

E12.2 [13P] Topics: *inf-convolution*. Difficulty: \*. When  $(X, d)$  is a metric space, and  $f : X \rightarrow \mathbb{R} \cup \{+\infty\}$  is l.s.c. and bounded from below, let

$$f_n(x) \stackrel{\text{def}}{=} \inf_{y \in X} \{f(y) + nd(x, y)\}$$

be the *inf-convolution*. Show that the sequence  $f_n$  is an increasing sequence of Lipschitz functions with  $f_n(x) \rightarrow_n f(x)$ .

**Solution 1.** [13Q]