Exercises

E12.2 [13R] Given $f : X \to \mathbb{R}$, define

$$f^*(x) = f(x) \vee \limsup_{y \to x} f(y) \quad ;$$

show that $f^*(x)$ is the smallest upper semicontinuous function that is greater than or equal to f at each point.

Similarly, define

$$f_*(x) = f(x) \wedge \liminf_{y \to x} f(y)$$

then $-(f^*) = (-f)_*$, and therefore $f_*(x)$ is the greatest lower semicontinuous function that is less than or equal to f at each point.

Finally, note that $f^* \geq f_*$.

Solution 1. [13S]