

Exercises

E12.11 [13T] Topics: oscillation.

Given any $f : X \rightarrow \mathbb{R}$, we define *oscillation function* $\text{osc}(f)$

$$\text{osc}(f)(x) \stackrel{\text{def}}{=} f^*(x) - f_*(x)$$

1. Note that $\text{osc}(f) \geq 0$, and that f is continuous in x if and only if $\text{osc}(f)(x) = 0$.
2. Show that $\text{osc}(f)$ is upper semicontinuous.
3. If (X, d) is a metric space, note that

$$\text{osc}(f)(x) \stackrel{\text{def}}{=} \lim_{\varepsilon \rightarrow 0^+} \sup\{|f(y) - f(z)|, d(x, y) < \varepsilon, d(x, z) < \varepsilon\}$$

Solution 1. [13V]