Exercises

E13.a.12 [13W] Let (X,τ) be a topological space and $f : X \to \mathbb{R}$ a function. Let $\overline{x} \in X$ be an accumulation point. Let eventually U_n be a family of open neighbourhoods of $\overline{x}$ with $U_n \supseteq U_{n+1}$. Then there exists a sequence $(x_n) \subset X$ with $x_n \in U_n$ and $x_n \neq \overline{x}$ and such that

$$\lim_{n\to\infty} f(x_n) = \liminf_{x\to\overline{x}} f(x) \quad .$$

(Note that in general we do not claim neither expect that $x_n \to \overline{x}$).

Solution 1. *[13X]*