Exercises

E12.13 [13Y] Let (X, τ) be a topological space and $f : X \to \mathbb{R}$ a function; let $\overline{x} \in X$ be an accumulation point; let A be the set of all the limits $\lim_n f(x_n)$ (when they exist) for all sequences $(x_n) \subset X$ such that $x_n \to \overline{x}$; then

$$\liminf_{x \to \overline{x}} f(x) \leq \inf A \; ;$$

moreover, if (X, τ) satisfies the first axiom of countability, then equality holds and $\inf A = \min A$.