Exercises

E12.18 [144] Let $I = [a, b]$. Let V be the set of functions $f : [a, b] \to \mathbb{R}$ that are piecewise constant; it is the vector space generated by $\mathbb{1}_J$, all the characteristic functions of all intervals $J \subseteq I$. Prove that the closure of V (according to uniform convergence) coincides with the space of regulated functions.

So the space of regulated functions, endowed with the norm $\|\cdot\|_\infty$, is a Banach space.