Definition 13.15. [155] *Let $A \subseteq \mathbb{R}$ and $f : A \to \mathbb{R}$ be a function; f is called* ***uniformly continuous*** *if*

$$\forall \varepsilon > 0,\ \exists \delta > 0,\ \forall x, y \in A,\ |x - y| < \delta \Longrightarrow |f(x) - f(y)| < \varepsilon\ .$$

More in general, given (X_1, d_1) and (X_2, d_2) metric spaces, given the function $f : X_1 \to X_2$, f is ***uniformly continuous*** *if*

$$\forall \varepsilon > 0,\ \exists \delta > 0,\ \forall x, y \in X_1,\ d_1(x, y) < \delta \Longrightarrow d_2(f(x), f(y)) < \varepsilon\ .$$