Exercises

E13.20 [15F] Difficulty:*. Let (X_1, d_1) and (X_2, d_2) metric spaces, with (X_2, d_2) complete. Let $A \subset X_1$ and $f : A \to X_2$ be a uniformly continuous function. Show that there is a uniformly continuous function $g : \overline{A} \to X_2$ extending f; In addition, the extension g is unique.

Note that if ω is a continuity modulus for f then it is also a continuity modulus for g. (We assume that ω is continuous, or, at least, that it is upper semicontinuous).

Solution 1. [15G]