Exercises

E13.29 [161]Prerequisites:[OPT]. Let (X_1, d_1), (X_2, d_2) and (Y, δ) be three metric spaces; consider the product $X = X_1 \times X_2$ equipped with the distance $d(x, y) = d_1(x_1, y_1) + d_2(x_2, y_2)$. [a] Let $f : X \to Y$ be a function with the following properties:

- For each fixed $x_1 \in X_1$ the function $x_2 \mapsto f(x_1, x_2)$ is continuous (as a function from X_2 to Y);

- There is a continuity modulus ω such that

  $$\forall x_1, \tilde{x}_1 \in X_2 \ , \ \forall x_2 \in X_2 \ , \delta\big(f(x_1, x_2), f(\tilde{x}_1, x_2)\big) \leq \omega\big(d_1(x_$$

  (We could define this property by saying that the function $x_1 \mapsto f(x_1, x_2)$ is uniformly continuous, with constants independent of the choice of x_2).

Then show that f is continuous.

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[a]We know from [109] and [OPT] that there are several possible choices of distances, but they are equivalent to each other.