

Exercises

13.37 [16S] Prerequisites: [2CX]. Difficulty: *.

Suppose (X, d) admits a subset D that is dense but has empty interior.^a

Given a $E \subset X$ which is a F_σ , construct a function $f : X \rightarrow \mathbb{R}$ for which E is the set of points of discontinuity.

Solution 1. [16T]

^aThat is, both D and the complement $X \setminus D$ are dense. $X = \mathbb{R}$ meets this requirement, taking as an example $D = \mathbb{Q}$.