

Exercises

E14.4 [16Z] Topics:simplex.

Given $x_0, \dots, x_k \in \mathbb{R}^n$, let

$$\left\{ \sum_{i=0}^k x_i t_i : \sum_{i=0}^k t_i = 1 \forall i, t_i \geq 0 \right\} \quad (14.4)$$

the set of all possible combinations: prove that this set is convex.

When the vectors $x_1 - x_0, x_2 - x_0 \dots x_k - x_0$ are linearly independent, the set defined above is a *simplex* of dimension k .

Show that, if $n = k$, then the simplex has a non-empty interior, equal to

$$\left\{ \sum_{i=0}^n x_i t_i : \sum_{i=0}^n t_i = 1 \forall i, t_i > 0 \right\} \quad (14.5)$$