Exercises

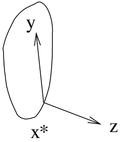
E14.15 [17D] Topics:projection.Difficulty:*. Note:*This is the well-known "projection theorem", which holds for A convex closed in a Hilbert space; if $A \subset \mathbb{R}^n$ then the proof is simpler, and it's a useful exercise..*

Given $A \subset \mathbb{R}^n$ closed convex non-empty and $z \in \mathbb{R}^n$, show that there is only one minimum point x^* for the problem

$$\min_{x \in A} \|z - x\| \ .$$

Show that x^* is the minimum if and only if

$$\forall y \in A, \langle z - x^*, y - x^* \rangle \leq 0 \ \ .$$



x^* is called "the projection of z on A".

(Note that this last condition is simply saying that the angle must be obtuse.)

Solution 1. *[17G]*