Exercises

E14.16 [17H] Topics:separation. Prerequisites:[17D].

Given $A \subset \mathbb{R}^n$ closed non-empty convex and $z \notin A$, let x^* be defined as in the previous exercise [17D]; define $\delta = \|z - x^*\|$, $v = (z - x^*)/\delta$ and $a = \langle v, x^* \rangle$. Prove that v, a and $v, a + \delta$ define two parallel hyperplanes that strongly separate z from A, in the sense that $\langle z, v \rangle = a + \delta$ but $\forall x \in A, \langle x, v \rangle \leq a$.