

Exercises

E14.18 [17M] Prerequisites: [17J], [11T]. If A, B are disjoint convex, with A open, show that there is a hyperplane separating A and B , that is, there exist $v \in \mathbb{R}^n$, $v \neq 0$ and $c \in \mathbb{R}$ such that

$$\forall x \in A, \langle x, v \rangle < c \text{ but } \forall y \in B, \langle y, v \rangle \geq c; \quad (14.19)$$

moreover show that if also B is open, then you can have strict separation (i.e. strict inequality in the last term in (14.19)).

(Hint: given $A, B \subseteq \mathbb{R}^n$ convex nonempty, show that

$$A - B \stackrel{\text{def}}{=} \{x - y, x \in A, y \in B\}$$

is convex; show that if A is open then $A - B$ is open, as in [11T].)

Solution 1. [17N]