Exercises

14.22 [180] Let $C \subset \mathbb{R}^n$ be a convex set. Let $f : C \to \mathbb{R}$ be convex; let $x_1, \dots, x_n \in C$ and $t_1, \dots, t_n \in [0, 1]$ be such that $\sum_{i=1}^{n} t_i = 1$. Show that

$$\sum_{i=1}^{n} t_i x_i \in C$$

and

$$f\left(\sum_{i=1}^{n} t_i x_i\right) \leq \sum_{i=1}^{n} t_i f(x_i) \,.$$