Exercises

E14.28 [182] Let $C \subseteq \mathbb{R}^n$ be a convex set, and $f : C \to \mathbb{R}$ a convex function. Given $l \in \mathbb{R}$, define the *sublevel set* as

$$L_l = \{x \in \mathbb{R}^n : f(x) \leq l\} \quad .$$

Show that L_l is a convex (possibly empty) set. Deduce that the minimum points of f are a convex (possibly empty) set. Show that if f is strictly convex there can be at most one minimum point.