Exercises

14.25 [188] Topics:subdifferential.Prerequisites:[184].Difficulty:*.

Let $C \subseteq \mathbb{R}^n$ be an open convex set, and $f : C \to \mathbb{R}$ a convex function; Given $z \in C$, we define the *subdifferential* $\partial f(z)$ as the set of v for which the relation [(14.24)] is valid (i.e., $\partial f(z)$ contains all vectors v defining the support planes to f in z).

$\partial f(z)$ enjoys interesting properties.

- $\partial f(z)$ is *locally bounded*: if $z \in C$ and $r > 0$ is such that $B(z, 2r) \subset C$, then $L > 0$ exists such that $\forall y \in B(z, r)$, $\forall v \in \partial f(x)$ you have $|v| \le L$. In particular, for every $z \in C$, we have that $\partial f(z)$ is a bounded set.
- Show that ∂f is *upper continuous* in this sense: if $z \in C$ and $(z_n)_n \subset C$ and $v_n \in \partial f(z_n)$ and if $z_n \to_n z$ and $v_n \to_n v$ then $v \in \partial f(z)$. In particular, for every $z \in C$, $\partial f(z)$ is a closed set.

Solution 1. *[189]*