Exercises

E15.d.3 [192] Let $f : [0, \infty) \to \mathbb{R}$ be concave, with $f(0) = 0$ and f continuous in zero.

- Prove that f is *subadditive*, i.e.

  $$f(t) + f(s) \geq f(t + s)$$

  for every $t, s \geq 0$. If moreover f is strictly concave and $t > 0$ then

  $$f(t) + f(s) > f(t + s) \,.$$

- Prove that, if $\forall x, f(x) \geq 0$, then f is weakly increasing.
- The other way around? Find an example of $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, continuous, monotonic increasing and subadditive, but not concave.

Solution 1. *[193]*